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PORTFOLIO SELECTION IN THE YIELD AND SEMI-ABSOLUTE DEVIATION SPACE WITH THE INCORPORATION OF AN ENVIRONMENTAL INDICATOR

Abstract:
Different portfolio models can be used to determine an investor's optimal strategy, diversifying assets to reduce the overall risk of the investment under consideration. In general, the main objectives of portfolio models can be identified as minimised risk and maximised return. In minimizing risk, an investor can use different risk measures, and in the paper presented, investment risk is considered to be the semi-absolute deviation from the expected return. Since the focus is now on sustainable investing, the model considered is extended to include environmental burden conditions on the investment based on the E indicator from the ESG indicators concept. It is generally assumed that implementing environmental criteria may be associated with increased risk. However, for various reasons, sustainable investing may be attractive to the investor, and the risk difference between conventional and sustainable investing may not be significant. The authors make a possible application of the above approach to the assets of the Dow Jones Industrial Average (DJIA) stock index. The paper aims to address the question of to what extent the incorporation of an environmental indicator in portfolio selection models affects the investor's risk, specifically for models based on semi-absolute deviation. The results of the calculations on DJIA index assets confirm that the calculated risk is higher, but the difference is not dramatic.
Keywords: portfolio, environmental, optimization

1. INTRODUCTION

The requirement for environmental sustainability of investments is increasingly encountered when making investment portfolio selection decisions (e.g. Fabozzi et al., 2002). It is generally assumed that implementing environmental criteria may be associated with increased risk. Nevertheless, for various reasons (environmental protection, reduction of greenhouse gases in the air, use of various ecological subsidies, etc.), investors are increasingly interested in investing in companies that meet environmental standards.
Green investing (e.g. Renneboog et al., 2008; Statman, 2000) generally refers to the search for investment opportunities that contribute to protecting and improving the environment. Such investing aims to support companies with a positive impact on environmental indicators, which are part of the ecological factor within the concept of ESG criteria (Environmental, Social, Governance).
In the first part of the paper, a model of portfolio selection based on a measure of risk measured by the semi-absolute deviation is presented, and this model is extended to include an environmental condition on the investment. Subsequently, an analysis is carried out by applying the presented model to the assets of the DJIA stock index (Dow Jones Industrial

Average) to investigate the impact of considering environmental criteria on the riskiness of the calculated portfolio. The results show an increase in risk when the environmental condition is considered, i.e. whether the additional environmental criterion significantly affects the risk-return ratio.
The contribution of the paper is oriented towards the extension of the portfolio selection model to the environmental indicator (E), which is currently a significant indicator of sustainable development based on the ESG concept (e.g. Varmaz et al.2024; Siqi, 2024). Younger investors, especially, are willing to take higher risks when investing in order to take into account the environmental aspect of the investment (of course, the set of efficient solutions is shrinking in the process). Based on portfolio selection models, it can be concluded that at the minimum return set by the investor (E_p), the difference between the risk with and without considering the environmental indicator E under consideration decreases as its value increases (Table 1).

2. PORTFOLIO SELECTION MODEL AUGMENTED WITH AN ENVIRONMENTAL INDICATOR

In 1999, Ogryczak and Ruszczynski (Ogryczak and Ruszczynski, 1999) introduced a model based on a model using absolute deviance (Konno and Yamazaki, 1991). This model uses the mean semi-absolute deviation to measure risk. The semi-absolute deviation is a way of measuring risk when the actual rate of return in the portfolio is less than the absolute value of the average rate of return in the portfolio, the objective being to minimize the same risk measure function as in the absolute deviation model.
The main benefits of their work include:
Use of semi-absolute variance—This method is more robust to outliers than classical variance (Markowitz, 1952), thus better capturing the risk of asymmetric return distributions.
Portfolio optimization - The approach presented by Ogryczak and Ruszczynski (Ogryczak & Ruszczynski, 1999; Belabbes & Guennoun, 2025) allows for an efficient solution to portfolio selection problems, considering investors' risk preferences.
Link with linear programming - Models based on semi-absolute deviation are formulated to be computationally efficient and well tractable using linear optimization methods (e.g., Fang and Wang, 2006).
Let r_{jt} be the *t-th* ($t=1,2,...T$, where T is the number of periods used in the construction of the model) component of the discrete random variable X_j represented by the vector $\mathbf{r}_j$ for j = 1, 2,...n, whose values are assumed to be available, whether they are historical returns or simulated return values. It is also assumed that the expected value of the random variable X_j can be expressed as a geometric mean derived from the data used, assuming an equal probability of occurrence of each state. Then the expected return values of the random variable X_j, j = 1, 2,...n, can be expressed:

$$E_j = E\left(\mathbf{r}_j\right) = \left(\prod_{t=1}^{T}\left(1+r_{jt}\right)\right)^{\frac{1}{T}} - 1 \tag{1}$$

If w_j represents the share of *the j-th* asset, j = 1, 2,...n, in the total investment, the expected return function of the portfolio can be written:

$$E(\mathbf{w}) = \sum_{j=1}^{n} E_j w_j \tag{2}$$

The goal of every investor is, of course, to maximize the value of the expected return on the investment $E(\mathbf{w})$ and, at the same time, to keep the risk of the investment as low as possible. In the above model, the risk is the semi-absolute deviation, which can be expressed as a risk function $\delta_-(\mathbf{w})$, where δ_- respects that the current rate of return in the portfolio is less than the absolute value of the average rate of return in the portfolio:

$$\delta_-(\mathbf{w}) = E\left[\left|\sum_{j=1}^{n}\mathbf{r}_j w_j - E\left[\sum_{j=1}^{n}\mathbf{r}_j w_j\right]\right|_-\right] \tag{3}$$

Then, given a minimum investor required return E_p based on the above risk measure, where the variables y_t represent the absolute deviation in each period t, $t=1,2,...T$ (T is the total number of states) from the expected value of the return, a portfolio selection model can be formulated based on the risk measure measured by the semi-absolute deviation (SAD):
Input parameters:
T - number of periods used in the construction of the model,
n - number of assets,
$\delta_-(\mathbf{w})$ - A risk function in which δ_- respects that the current rate of return in the portfolio is less than the absolute value of the average rate of return in the portfolio,
r_{jt} - the *t-th* component of the discrete random variable X_j, j = 1, 2,...n, $t=1,2,...T$,
E_j - values of the expected return of the random variable X_j, j = 1, 2,...n,
w_j - the share of *the j-th* asset in the total investment, j = 1, 2,...n
E_p - *the* minimum return required by the investor.

Variables:
w_j - the share of *the j-th* asset, j = 1, 2,...n, in the total investment,
y_t - absolute deviation from the expected return value in each period t, $t=1,2,...T$ (T is the total number of states).

$$
\begin{aligned}
&\min \delta_{-}(\mathbf{w},\mathbf{y}) = \frac{1}{T}\sum_{t=1}^{T}\frac{y_t - \sum_{j=1}^{n}\left(r_{jt}-E_j\right)w_j}{2} \\
&y_t + \sum_{j=1}^{n}\left(r_{jt}-E_j\right)w_j \geq 0, \quad t = 1,2,...T \\
&y_t - \sum_{j=1}^{n}\left(r_{jt}-E_j\right)w_j \geq 0, \quad t = 1,2,... \\
&\sum_{j=1}^{n} E_j w_j \geq E_P \\
&\sum_{j=1}^{n} w_j = 1 \\
&w_1, w_2, ... w_n \geq 0
\end{aligned}
\tag{4}
$$

In sustainable investing, it is desirable to include the values of the environmental indicator under consideration in the model, which will meet the investor's requirement to consider only those assets that do not exceed its specific desired value (Shushi, 2023). In the analysis carried out, model (4) was extended to include indicator E from the ESG concept (e.g. Friede et al., 2015). The parameter used to build the model was: the desired value of E (EN_E) The input parameters were the values of E for each asset EN_j (indicator for the *j-th* asset, $j=1,2,...n$). Then, the model can be extended with the following condition:

$$
\sum_{j=1}^{n} EN_j w_j \leq EN_E \tag{5}
$$

In bound (5), the investor can individually determine the maximum value of the environmental indicator EN_E according to the demandingness he places on the values of E (e.g., on finance.yahoo.com the values of the rating agency Sustainalytics) for the assets under consideration (e.g., the values of the assets of the financial indices DJIA, S&P 500, DAX, and so on, but also the values of any assets under consideration). In further analysis, the value of the EN_E indicator was considered to be the boundary between the 1st and 2nd quartile of DJIA assets (i.e., 25% of the smallest E values of all assets considered).
Clearly, introducing an additional condition (5) into model (4) reduces the set of efficient solutions. In practice, this means that only those efficient portfolios that satisfy condition (5) are computed in addition to the original bounds. However, since the parameter EN_E has been taken into account for all asset values EN_j ($j=1,2,...n$), stocks with a higher value of E but also with a higher return can be accepted into the portfolio. In the case of a strict requirement to meet the condition of the desired value of E, the calculations could only be performed on the relevant set of assets (in the considered case of 29 DJIA assets only for the seven assets with the lowest value of E) without condition (5).

3. ANALYSIS OF THE INTRODUCTION OF ENVIRONMENTAL CONDITIONS

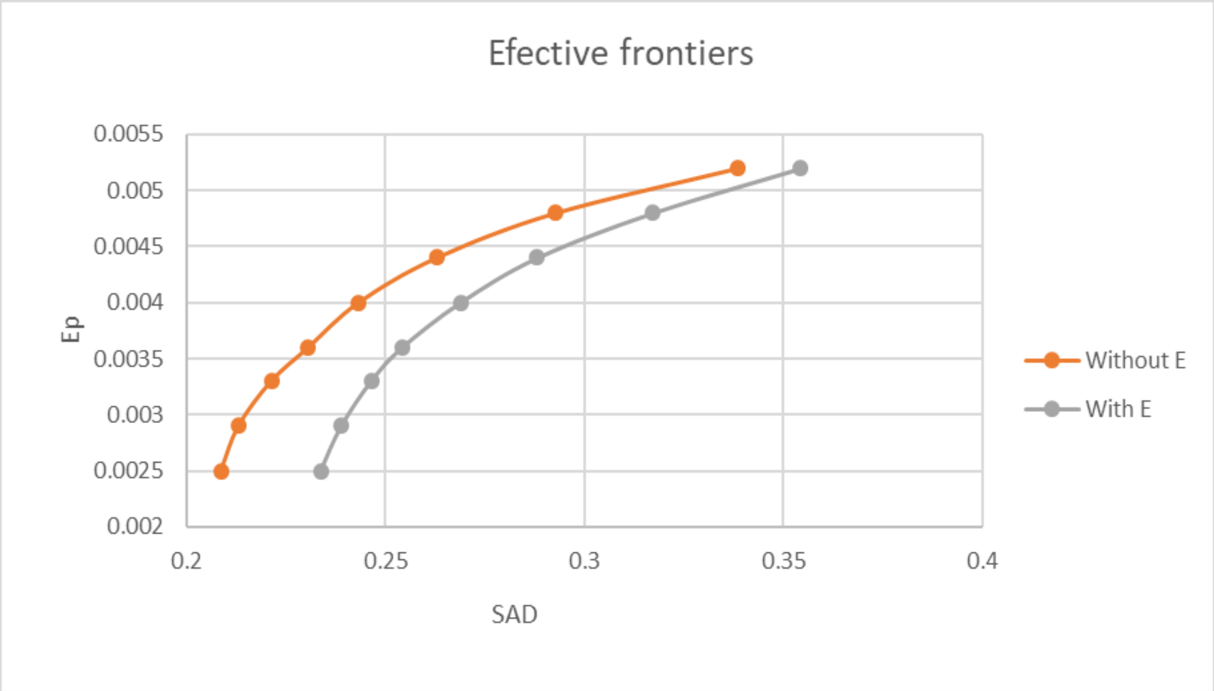
For a good application of the presented model, it is important to correctly determine the time period for the analysis of historical stock prices. This time span is crucial to obtain relevant data on which the average return is then calculated. In the case under consideration, the evolution of the weekly prices of 29 stocks of the DJIA index over a specific period, namely from January 1, 2020 to December 31, 2024, was monitored.
In addition to stock price performance, ESG metrics for selected DJIA index stocks should be obtained. The relevant values are available from the rating agency Sustainalytics at finance.yahoo.com (Sustainalytics, 2024). The agency typically publishes ESG ratings on an annual basis, and the environmental indicator data for 2023 was used in the analysis. In the solution, the maximum weighted value of the environmental indicator for the portfolio was set as the value of the boundary between the first and second quartile, which takes the value of 0.9 for the above stocks.
Portfolio optimization based on a portfolio selection model based on a risk measure measured by the semi-absolute deviation (SAD) accepting the investment sustainability condition was performed using Python. The results (efficient portfolios) are shown in Table 1 in the E section. Calculations were also performed without considering the E condition to compare the impact of the sustainability conditions (Table 1 part without E).

Table 1 Calculated efficient portfolios with and without environmental condition

Portfolio	Ep	SAD with E	SAD without E	Increase in SAD
P1	0.25%	23.38%	20.87%	12.0%
P2	0.29%	23.90%	21.31%	12.2%
P3	0.33%	24.65%	22.13%	11.4%
P4	0.36%	25.43%	23.06%	10.3%
P5	0.40%	26.89%	24.32%	10.6%
P6	0.44%	28.82%	26.28%	9.7%
P7	0.48%	31.72%	29.26%	8.4%
P8	0.52%	35.44%	33.84%	4.7%

Source: Author's *calculations*

A graphical interpretation of the results is shown in Figure 1.



Picture 1 Effective boundary in case of E boundary and without E boundary

Source: Author's *calculations*

Table 1 and Figure 1 show that as the value of the minimum investor required rate of return E_p increases, the difference between the calculated values without and with the environmental requirement (5) decreases. Thus, it is clear that as the environmental aspect decreases, the risk/expected return ratio for portfolios also decreases. Such an approach with the application of the environmental aspect can provide the investor with a basis on which to determine his portfolio construction strategy.

It is also interesting to compare the composition of the individual portfolios. Table 2 shows the individual returns of assets that occurred at least once in the efficient portfolio. Columns P1 to P8 report the individual stock weights for the respective efficient portfolio. Similarly, Table 3 provides information on stock returns and weights for the efficient portfolios without considering the environmental benchmark.

Table 2 Efficient portfolios taking environmental conditions into account.

	Retrieved from	P1	P2	P3	P4	P5	P6	P7	P8
AAPL	0.56%	12.5%	16.8%	21.6%	25.7%	29.3%	34.5%	44.9%	61.8%
AMGN	0.15%	3.9%	0.8%	3.9%	5.7%	1.8%	0.0%	0.0%	0.0%
AXP	0.50%	1.4%	2.1%	3.4%	5.0%	6.1%	10.8%	9.6%	6.2%
CSCO	0.20%	8.9%	6.3%	1.5%	0.0%	0.0%	0.0%	0.0%	0.0%
DIS	-0.01%	4.2%	2.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GS	0.50%	0.0%	0.0%	0.0%	0.0%	2.9%	2.7%	8.5%	16.7%
IBM	0.36%	5.7%	7.1%	9.1%	12.4%	15.6%	12.9%	6.1%	0.0%
JNJ	0.08%	24.6%	23.4%	15.2%	6.1%	0.0%	0.0%	0.0%	0.0%
JPM	0.35%	4.1%	4.6%	3.3%	2.9%	0.0%	0.0%	0.0%	0.0%
MRC	0.15%	12.8%	12.4%	12.6%	14.0%	13.6%	5.5%	0.0%	0.0%
MSFT	0.46%	2.9%	7.9%	6.3%	4.9%	6.1%	7.3%	6.5%	2.4%
TRV	0.33%	4.9%	4.1%	8.4%	7.8%	7.9%	7.1%	2.7%	0.0%
UNH	0.33%	10.3%	12.4%	14.7%	15.6%	16.7%	16.2%	15.7%	6.1%
V	0.27%	3.8%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
WMT	0.39%	0.0%	0.0%	0.0%	0.0%	0.0%	3.0%	6.0%	6.8%

Source: Author's *own calculations*

Table 3 Efficient portfolios without considering the environmental condition

	Wine	P1	P2	P3	P4	P5	P6	P7	P8
AAPL	0.56%	0.0%	1.1%	3.6%	7.4%	13.5%	21.0%	35.5%	59.1%
AXP	0.50%	0.0%	0.0%	0.0%	0.0%	0.0%	0.6%	7.4%	3.7%
CAT	0.48%	1.3%	2.5%	3.5%	4.2%	6.8%	8.1%	18.1%	24.8%
CRM	0.42%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
CVX	0.26%	2.8%	2.6%	2.4%	1.6%	0.0%	0.0%	0.0%	0.0%
DIS	-0.01%	2.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GS	0.50%	0.0%	0.0%	0.0%	0.0%	2.3%	5.0%	0.0%	2.7%
IBM	0.36%	5.6%	9.4%	11.7%	13.2%	13.5%	12.6%	0.0%	0.0%
JNJ	0.08%	10.8%	5.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
JPM	0.35%	0.0%	2.6%	3.5%	4.3%	4.1%	0.0%	0.0%	0.0%
KO	0.16%	0.0%	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
MCD	0.23%	15.4%	10.9%	10.1%	7.1%	0.3%	0.0%	0.0%	0.0%
MMM	0.10%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
MRC	0.15%	10.5%	8.1%	12.1%	11.5%	8.6%	0.0%	0.0%	0.0%
MSFT	0.46%	12.4%	12.0%	10.7%	13.4%	10.0%	8.3%	7.7%	0.0%
PG	0.20%	14.8%	15.1%	10.2%	3.1%	0.0%	0.0%	0.0%	0.0%
UNH	0.33%	2.1%	3.3%	4.5%	3.0%	1.7%	1.8%	0.0%	0.0%
VZ	-0.01%	6.9%	6.7%	2.2%	0.0%	0.0%	0.0%	0.0%	0.0%
WMT	0.39%	15.2%	20.2%	25.4%	31.3%	39.2%	42.6%	31.4%	9.7%

Source: Author's *calculations*

In both cases, the following actions occurred: AAPL, AXP, DIS, GS, IBM, JNJ, JPM, MRK, MSFT, UNH, and WMT. In contrast to the portfolio without considering the environmental criterion, where the main weight is on AAPL and WMT, when the criterion is considered, the main weight is shifted to AAPL due to its low environmental indicator value (0.5) The calculated weight values of the considered stocks w_j ($j=1,2,...n$), with (or without) the application of the environmental aspect, can again provide the investor with the basis on which to determine his own strategy for creating the portfolio.

4. CONCLUSION

The paper focuses on the analysis of the impact of incorporating environmental criteria into the portfolio selection model. The environmental constraint was implemented through a structural constraint that sets a maximum value of the weighted average of the portfolio's environmental score at the boundary between the first and second quartile.
An analysis of the Dow Jones Industrial Average (DJIA) stock index compared the differences between portfolios constructed with and without the environmental constraint to assess the impact of environmental criteria on portfolio performance.
The results of the analysis on the Dow Jones Industrial Average (DJIA) stock index showed that the consideration of environmental criteria influences the selection of assets in the portfolio and its resulting performance. Key findings include:
Higher portfolio riskiness—The inclusion of the environmental factor led to an increase in the semi-absolute deviation, indicating a higher portfolio risk compared to portfolios without this constraint.
Changing asset weights: Companies with lower environmental scores received a higher representation in the optimised portfolio, while the overall investment structure also changed.
Based on these findings, it can be concluded that environmental factors play a significant role in portfolio selection. Their inclusion may lead to a change in an investment's risk-return profile, but at the same time, it provides investors with the opportunity to allocate capital to more sustainable assets.
The analyzed problem of extending the portfolio selection model using absolute deviation with an environmental indicator (E) and its impact on the increase in risk at the minimum return set by the investor determined the structure of the paper. After formulating the problem, the first part of the paper presents the essence of the portfolio selection model, which is based on a measure of risk measured by the semi-absolute deviation. In contrast, this model is extended to include the environmental condition of the investment. In the third part of the paper, an analysis is carried out by applying the presented model to the assets of the DJIA stock index (Dow Jones Industrial Average) in order to investigate the impact of considering the environmental criteria on the riskiness of the computed portfolio. The performed calculations show an increase in risk when the environmental condition is considered, i.e. whether the additional environmental criterion significantly affects the risk-return ratio. They show that, at the minimum return set by the investor (E_p), the difference between the risk when considering the considered environmental indicator E and without considering it decreases as its value increases (Table 1).

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